

# Spin Triplet Superconductivity in $\text{Sr}_2\text{RuO}_4$ due to Orbital and Spin Fluctuations: Analysis by Two-Dimensional Renormalization Group Theory

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We study the mechanism of the triplet superconductivity in  $\text{Sr}_2\text{RuO}_4$  based on the multi-orbital Hubbard model. The electronic states are studied using the renormalization group method. Thanks to the vertex correction (VC) for the susceptibility, which is dropped in the mean-field-level approximations, strong orbital and spin fluctuations at  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$  emerge in the quasi one-dimensional Fermi surfaces composed of  $d_{xz}$  and  $d_{yz}$  orbitals. Due to the cooperation of both fluctuations, we obtain the triplet superconductivity in the  $E_u$  representation, in which the superconducting gap is given by the linear combination of  $(\Delta_x(\mathbf{k}), \Delta_y(\mathbf{k})) \sim (\sin 3k_x, \sin 3k_y)$ . These results are confirmed by a diagrammatic calculation called the self-consistent VC method.

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$\text{Sr}_2\text{RuO}_4$  is an unconventional superconductor with the transition temperature  $T_c = 1.5\text{K}$  [1–3]. This material has been attracting great attention since the spin triplet superconductivity (TSC) is indicated by the NMR measurements [4]. From the early stage, the chiral  $p$ -wave ( $p_x + ip_y$ ) TSC, which is analogous of the A-phase of the superfluid  $^3\text{He}$ , had been predicted [5]. However, in contrast to the paramagnon mechanism in  $^3\text{He}$ , no ferro-magnetic fluctuations are observed in  $\text{Sr}_2\text{RuO}_4$ . Instead, strong antiferro-magnetic (AFM) fluctuations with  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$  are observed by neutron scattering spectroscopy [6]. Since the AFM fluctuations give the spin singlet superconductivity (SSC) in usual, the mechanism of the TSC in  $\text{Sr}_2\text{RuO}_4$  has been a long-standing problem in strongly correlated electron systems.

Figures 1 (a) and (b) show the bandstructure and the Fermi surfaces (FSs) of  $\text{Sr}_2\text{RuO}_4$ : The quasi-one-dimensional (q1D) FSs,  $\text{FS}\alpha$  and  $\text{FS}\beta$ , are composed of ( $d_{xz}$ ,  $d_{yz}$ )-orbitals, and the nesting of these q1D FSs is the origin of the AFM fluctuations at  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$ . The two-dimensional (2D) FS,  $\text{FS}\gamma$ , is composed of only  $d_{xy}$ -orbital. If the spin-orbit interaction (SOI) is neglected, the ( $\alpha, \beta$ )-bands and  $\gamma$ -band are coupled only via the electron-electron correlation. Therefore, the superconductivity would be realized mainly in either the q1D bands ( $|\Delta_{\alpha, \beta}| \gg |\Delta_\gamma|$ ) or the 2D band ( $|\Delta_{\alpha, \beta}| \ll |\Delta_\gamma|$ ).

The mechanisms of the TSC originating mainly from the 2D band had been proposed in Refs. [7, 9–11]: Nomura and Yamada studied the TSC state using the perturbation theory [7], which is the natural development of the Kohn-Luttinger mechanism [8]. Recently, a three-orbital Hubbard model had been studied using a 2D renormalization group (RG) method [9]. They obtained the  $p$ -wave gap on the  $\text{FS}\gamma$  accompanied by the development of spin fluctuations at  $\mathbf{q} = (0.19\pi, 0.19\pi)$ . Also, charge-fluctuation-mediated TSC was discussed by introducing the inter-site Coulomb interaction [10].

On the other hand, one may expect that the TSC is

closely related to the AFM fluctuations in the q1D FSs at  $\mathbf{q} \sim \mathbf{Q}$ . The TSC originating from the q1D FSs had been discussed by applying the perturbation theory [12] and random-phase-approximation (RPA) [13, 14]. Takimoto discussed the orbital-fluctuation-mediated TSC using the RPA under the condition  $U' > U$ , where  $U$  ( $U'$ ) is the intra-orbital (inter-orbital) Coulomb interaction [13]. However, in the RPA, the SSC is obtained under the realistic condition  $U \geq U'$  due to strong AFM fluctuations. The TSC due to ferro-charge fluctuations was also discussed [15]. When the spin fluctuation is Ising-like, the TSC may be favored since the pairing interaction for the SSC is reduced [14]. In these studies, however, it is difficult to obtain the TSC based on the realistic multi-orbital Hubbard model, under the existence of strong AFM fluctuations as in  $\text{Sr}_2\text{RuO}_4$ .

To find out the origin of the TSC in  $\text{Sr}_2\text{RuO}_4$ , many experimental efforts have been devoted to determine the gap structure, such as the tunnel junction [18], ARPES, and quasiparticle interference measurements. Recently, large superconducting gap with  $2|\Delta| \approx 5T_c$  was observed by the scanning tunneling microscopy measurements [16]. The observed large gap would be that on the q1D FSs, since the tunneling will be dominated by the ( $d_{xz}$ ,  $d_{yz}$ )-orbitals that stand along the  $z$ -axis, as clarified in the double-layer compound  $\text{Sr}_3\text{Ru}_2\text{O}_7$  [17]. Therefore, it is an important challenge to establish the theory of the TSC based on the q1D-band Hubbard model, by applying an advanced theoretical method.

In this paper, we study the mechanism of the TSC in  $\text{Sr}_2\text{RuO}_4$  based on the realistic ( $U > U'$ ) two-orbital Hubbard model. The electronic states are studied using the 2D RG method developed in Ref. [19]. Thanks to the vertex correction (VC) for the susceptibility dropped in the RPA, strong orbital and spin fluctuations at  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$  emerge in the q1D bands [20]. We propose that the  $E_u$ -type TSC is realized by the cooperation of strong orbital and spin fluctuations in  $\text{Sr}_2\text{RuO}_4$ .

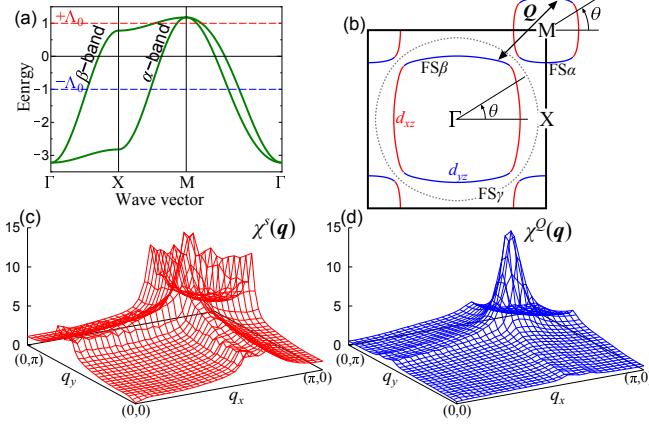


FIG. 1: (color online) (a) Bandstructure and (b) FSs of the two-orbital model.  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$  is the nesting vector. The FS $\gamma$  of Sr<sub>2</sub>RuO<sub>4</sub> is shown by dotted line. (c)  $\chi^s(\mathbf{q})$  and (d)  $\chi^Q(\mathbf{q})$  of the q1D-band model obtained by the RG+cRPA method ( $\Lambda_0 = 1$ ) for  $U = 3.5$ ,  $J/U = 0.035$  and  $T = 0.02$ .

In this paper, we study the two-orbital Hubbard model, which describes the quasi-1D FSs of Sr<sub>2</sub>RuO<sub>4</sub>. The kinetic term is given by  $H_0 = \sum_{\mathbf{k}, \sigma} \sum_{l,m} \xi_{\mathbf{k}}^{l,m} c_{\mathbf{k},l,\sigma}^\dagger c_{\mathbf{k},m,\sigma}$ , where the orbital indices  $l, m = 1$  and 2 refer to  $d_{xz}$ - and  $d_{yz}$ -orbitals, respectively. In the present model,  $\xi_{\mathbf{k}}^{1,1} = -2t \cos k_x - 2t_{nn} \cos k_y$ ,  $\xi_{\mathbf{k}}^{2,2} = -2t \cos k_y - 2t_{nn} \cos k_x$ , and  $\xi_{\mathbf{k}}^{1,2} = 4t' \sin k_x \sin k_y$ . Hereafter, we set  $(t, t_{nn}, t') = (1, 0.1, 0.1)$ , and fix the filling as  $n = 4 \cdot (2/3) = 2.67$ , which corresponds to the filling of the q1D FSs of Sr<sub>2</sub>RuO<sub>4</sub>. We also introduce the on-site Coulomb interactions  $U$ ,  $U'$ , and put the exchange and Hund's couplings  $J = J' = (U - U')/2$  throughout the paper.

Here, we analyze this model by applying the RG combined with the constrained RPA (RG+cRPA) [19]. This method is very powerful to calculate the higher-order many-body effects systematically and in an unbiased way. In the RG+cRPA method, we divide the lower-energy region ( $|E| < \Lambda_0$ ) of the Brillouin zone into  $N$  patches as done in Refs.[21–24] and perform the RG analysis. The contributions from the higher-energy region ( $|E| > \Lambda_0$ ) are calculated by the cRPA method with high numerical accuracy, and incorporated into the initial vertex functions [19]. (The conventional patch-RG method [21–24] is recovered when  $\Lambda_0 > W_{\text{band}}$ .) Although the initial vertex functions are very small, they play decisive roles for the fixed point of the RG flow.

We use  $N = 64$  (32 patches for each FS) in the present study, and it is verified that the results of  $N = 128$  are almost unchanged. First, we calculate the susceptibilities using the RG+cRPA: The charge (spin) susceptibility is given by  $\chi_{l,l',m,m'}^{(s)}(\mathbf{q}) = \int_0^\beta d\tau \frac{1}{2} \langle A_{l,l'}^{(s)}(\mathbf{q}, \tau) A_{m',m}^{(s)}(-\mathbf{q}, 0) \rangle e^{i\omega_l \tau}$ , where  $A_{l,l'}^{(s)}(\mathbf{q}) = \sum_{\mathbf{k}} (c_{\mathbf{k},l',\uparrow}^\dagger c_{\mathbf{k}+\mathbf{q},l,\uparrow} + (-) c_{\mathbf{k},l',\downarrow}^\dagger c_{\mathbf{k}+\mathbf{q},l,\downarrow})$ ,  $\mathbf{q} = (\mathbf{q}, \omega_l)$ , and

$l, l', m, m'$  are  $d$  orbitals. The quadrupole susceptibility with respect to  $O_{x^2-y^2} = n_{xz} - n_{yz}$  is given as  $\chi^Q(\mathbf{q}) = \sum_{l,m} (-1)^{l+m} \chi_{l,l',m,m'}^c(\mathbf{q})$ . Figures 1 (c) and (d) show the obtained  $\chi^s(\mathbf{q}) = \sum_{l,m} \chi_{l,l',m,m'}^s(\mathbf{q})$  and  $\chi^Q(\mathbf{q})$ , respectively, by the RG+cRPA method ( $\Lambda_0 = 1$ ) for  $U = 3.5$  and  $J/U = 0.035$  at  $T = 0.02$ . Both susceptibilities have the peak at  $\mathbf{Q} \approx (2\pi/3, 2\pi/3)$ , which is the nesting vector of the present FSs. The shape of  $\chi^s(\mathbf{q})$  is essentially equivalent to that of the RPA, by putting  $U = 2.2$  and  $J/U = 0.035$ . However,  $\chi^Q(\mathbf{q})$  in the RPA is quite small when  $J > 0$  [25, 26]. Therefore, the enhancement of  $\chi^Q(\mathbf{q})$  in Fig. 1 (d) originates from the many-body effect beyond the RPA. The natural candidate is the Aslamazov-Larkin (AL) type VC for  $\chi^Q(\mathbf{q})$ ,  $X^c(\mathbf{q})$ , whose analytic expression is given in Ref. [25]. Since  $X^c(\mathbf{q}) \sim U^4 T \sum_{\mathbf{k}} \Lambda_{\text{AL}}(\mathbf{q}; \mathbf{k})^2 \chi^s(\mathbf{k}) \chi^s(\mathbf{k} + \mathbf{q})$  for simplicity,  $X^c(\mathbf{q})$  takes large value at  $\mathbf{q} = \mathbf{0}$  and  $2\mathbf{Q}$  when  $\chi^s(\mathbf{k})$  is large at  $\mathbf{k} = \mathbf{Q}$ .  $\Lambda_{\text{AL}}(\mathbf{q}; \mathbf{k})$  is the three-point vertex composed of three Green functions [25]. In the present model,  $2\mathbf{Q} \approx \mathbf{Q}$  in the first Brillouin zone. Thus, with the aid of the VC and the nesting of the FSs, the enhancement of  $\chi^Q(\mathbf{Q})$  in Fig. 1 (d) is realized.

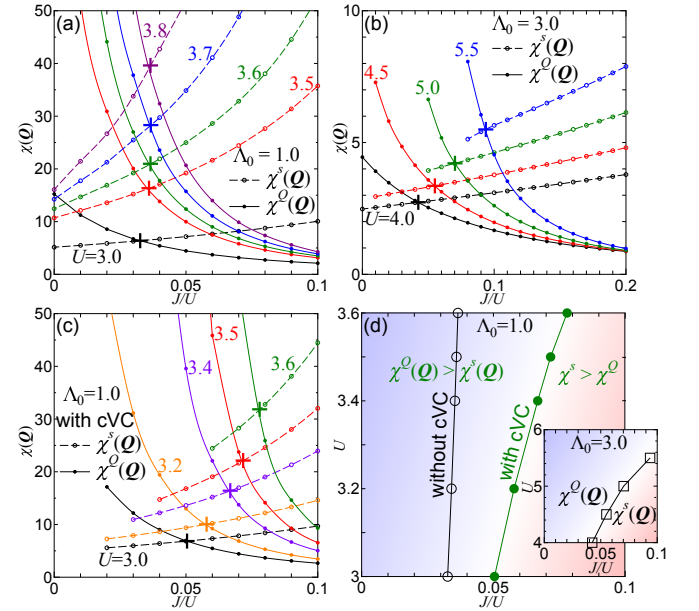


FIG. 2: (color online)  $\chi^s(\mathbf{Q})$  and  $\chi^Q(\mathbf{Q})$  as functions of  $J/U$  given by the RG+cRPA method for (a)  $\Lambda_0 = 1$  ( $U = 3.0 \sim 3.8$ ) and (b)  $\Lambda_0 = 3$  ( $U = 4.0 \sim 5.5$ ). (c)  $\chi^s(\mathbf{Q})$  and  $\chi^Q(\mathbf{Q})$  for  $\Lambda_0 = 1$ , by including the constrained VC (cVC). (d) Obtained phase diagram for  $\Lambda_0 = 1$  and  $\Lambda_0 = 3$  (inset).

Figure 2 (a) shows  $\chi^s(\mathbf{Q})$  and  $\chi^Q(\mathbf{Q})$  as functions of  $J/U$  at  $T = 0.02$ , obtained by the RG+cRPA method with  $\Lambda_0 = 1$ . For each value of  $U$ ,  $\chi^Q(\mathbf{Q})$  ( $\chi^s(\mathbf{Q})$ ) decreases (increases) with  $J/U$ , and they are equal at  $(J/U)_c \sim 0.035$ . We stress that  $(J/U)_c$  is negative in the RPA since the VC is totally dropped. In the case of  $\Lambda_0 = 3$  shown in Fig. 2 (b), the value of  $(J/U)_c$  increases

to  $\sim 0.08$  at  $U \sim 5$ , indicating that importance of the VC due to higher energy region. To check this expectation, we include the constrained AL term (cVC) in addition to the cRPA [19]. The obtained results are shown in Fig. 2 (c). It is verified that  $(J/U)_c$  increases to 0.08 at  $U = 3.7$ .  $\chi^Q(\mathbf{Q})$  in Fig. 2 (c) is approximately given by shifting  $\chi^Q(\mathbf{Q})$  in Fig. 2 (a) horizontally by  $+0.02 \sim +0.05$ . The values of  $(J/U)_c$  obtained by Figs. 2 (a)-(c) are summarized in Fig. 2 (d). Note that  $(J/U)_c \sim 0.1$  ( $\sim 0.15$ ) in the SC-VC $_{(\Sigma)}$  method [25, 27].

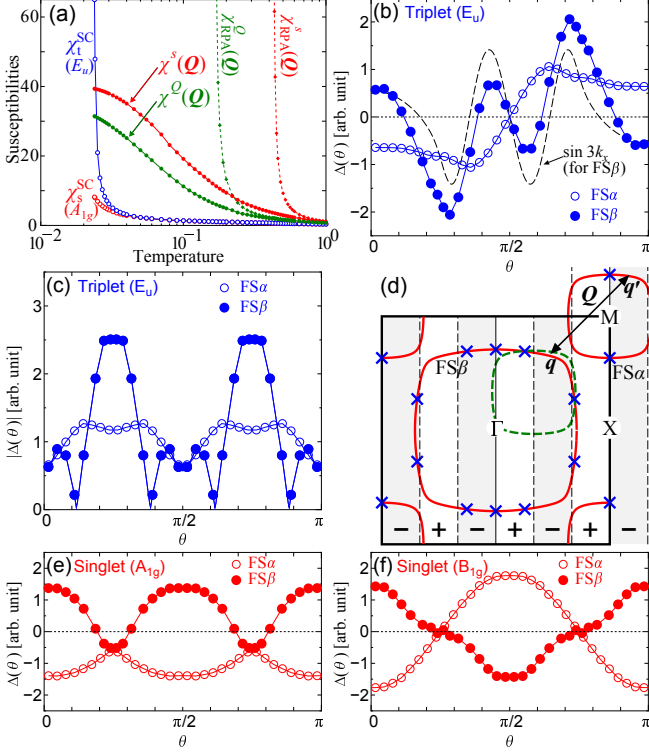


FIG. 3: (color online) (a)  $T$ -dependences of  $\chi^s(\mathbf{Q})$ ,  $\chi^Q(\mathbf{Q})$ ,  $\chi_{\text{t(s)}}^{\text{SC}}$  and  $\chi_{\text{t(s)}}^{\text{SC}}$  for  $U = 3.8$  and  $J/U = 0.04$  ( $\Lambda_0 = 1$ ). (b)  $E_u$  gap functions on FS $\mu$ ,  $\Delta_x^\mu(\theta)$  ( $\mu = \alpha, \beta$ ) obtained by the RG. The relation  $\Delta_x^\beta \propto \sin 3k_x$  holds approximately.  $N = 128$  patches are used. (c) The magnitude of the chiral (or helical) gap state  $|\Delta^\mu| = \sqrt{(\Delta_x^\mu)^2 + (\Delta_y^\mu)^2}$ . (d) Schematic explanation for the  $\sin 3k_x$ -type TSC due to orbital+spin fluctuations at  $\mathbf{q} = \mathbf{Q}$ . Solid lines (broken lines) are the necessary (accidental) nodes. The positions of nodes ( $\Delta_x^\mu = 0$ ) in (b) are shown by crosses. (e)  $A_{1g}$  and (f)  $B_{1g}$  SSC gap functions.

Although the value of  $(J/U)_c$  is underestimated at  $\Lambda_0 = 1$ , the obtained  $\chi^s(\mathbf{q})$  and  $\chi^Q(\mathbf{q})$  at  $\Lambda_0 = 1$  is reliable, since the higher-energy processes can be calculated with high numerical accuracy [19]. Hereafter, we perform the RG+cRPA method with  $\Lambda_0 = 1$ , by using smaller  $J/U$  ( $\sim 0.04$ ) to compensate for the absence of the higher-energy VCs. Figure 3 (a) shows the  $T$ -dependences of  $\chi^s(\mathbf{Q})$  and  $\chi^Q(\mathbf{Q})$  given by the RG+cRPA method ( $\Lambda_0 = 1$ ) for  $U = 3.8$  and  $J/U = 0.04$ : Both of them are strongly renormalized from the

RPA results. In the RPA,  $\chi_{\text{RPA}}^s(\mathbf{Q})$  diverges at  $T \approx 0.4$ , at which  $\chi_{\text{RPA}}^Q(\mathbf{Q})$  remains very small. In highly contrast, in the RG+cRPA method, the relation  $\chi^s(\mathbf{Q}) \approx \chi^Q(\mathbf{Q})$  holds for wide temperature range.

We also calculate the TSC and SSC susceptibilities using the RG+cRPA method:

$$\chi_{\text{t(s)}}^{\text{SC}} = \frac{1}{2} \int_0^\beta d\tau \langle B_{\text{t(s)}}^\dagger(\tau) B_{\text{t(s)}}(0) \rangle, \quad (1)$$

where  $B_{\text{t(s)}} = \sum_{\mathbf{q}, \mu} \Delta_{\text{t(s)}}^\mu(\mathbf{q}) c_{\mathbf{q}, \mu, \uparrow} c_{-\mathbf{q}, \mu, \uparrow(\downarrow)}$ .  $\mu = \alpha, \beta$  is the band index, and  $\Delta_{\text{t(s)}}^\mu(\mathbf{q})$  is the odd (even) parity gap function. The obtained  $\chi_{\text{t(s)}}^{\text{SC}}$  is shown in Fig. 3 (a), by optimizing the functional form of  $\Delta_{\text{t(s)}}^\mu(\mathbf{q})$  numerically [28]. Since  $\chi_{\text{t(s)}}^{\text{SC}}$  diverges at  $T = T_c$ , the strong development of  $\chi_{\text{t(s)}}^{\text{SC}}$  at  $T \approx 0.02$  means that the TSC is realized. This TSC state belongs to the two-dimensional  $E_u$ -representation,  $(\Delta_x^\mu(\mathbf{q}), \Delta_y^\mu(\mathbf{q}))$ . The obtained  $\Delta_x^\mu$  on the FSs when  $\chi_{\text{t(s)}}^{\text{SC}} \sim 60$  are shown in Fig. 3 (b), where  $\theta$  is the angle of the Fermi momentum shown in Fig. 1 (b). The necessary nodes  $\Delta_x^\mu = 0$  are on the lines  $q_{x(y)} = 0, \pm\pi$ . Very similar TSC gap is obtained for  $J/U \lesssim 0.08$  by taking the cVC into account with  $\Lambda_0 = 1$ . Below  $T_c$ , the BCS theory tells that the chiral or helical gap state with the gap amplitude  $|\Delta^\mu| = \sqrt{(\Delta_x^\mu)^2 + (\Delta_y^\mu)^2}$ , which is shown in Fig. 3 (c), is realized to gain the condensation energy.

To understand why the TSC state is obtained, it is useful to analyze the linearized gap equation:

$$\lambda_a^E \bar{\Delta}_a^\mu(\mathbf{q}) = - \sum_{\mu'}^{\alpha, \beta} \int_{\text{FS}_{\mu'}} \frac{d\mathbf{q}'}{v_{\mathbf{q}'}} V_a^{\mu, \mu'}(\mathbf{q}, \mathbf{q}') \bar{\Delta}_a^{\mu'}(\mathbf{q}') \times \ln(1.13\omega_c/T), \quad (2)$$

where  $a = \text{t or s}$ .  $\lambda_a^E$  is the eigenvalue,  $V_a^{\mu, \mu'}(\mathbf{q}, \mathbf{q}')$  is the pairing interaction, and  $\omega_c$  is the cut-off energy of the interaction. As shown in Fig. 1 (b), the inter-band interaction ( $\mu = \alpha, \mu' = \beta$ ) with  $\mathbf{q} - \mathbf{q}' = \mathbf{Q}$  is approximately given by the intra-orbital interaction given as

$$V_a^l(\mathbf{q}; \mathbf{q}') = b_a \frac{U^2}{2} |\Lambda_l^s(\mathbf{q}; \mathbf{q}')|^2 \chi_l^s(\mathbf{q} - \mathbf{q}') + c_a \frac{U^2}{2} |\Lambda_l^c(\mathbf{q}; \mathbf{q}')|^2 \chi_l^c(\mathbf{q} - \mathbf{q}'), \quad (3)$$

where  $(b_t, c_t) = (-1, -1)$  and  $(b_s, c_s) = (3, -1)$ , and  $\chi_l^{s,c}(\mathbf{Q}) \equiv \chi_{l,l;l,l}^{s,c}(\mathbf{Q})$ . (Note that  $\chi_l^s(\mathbf{Q}) \approx \chi^s(\mathbf{Q})/2$  and  $\chi_l^c(\mathbf{Q}) \approx \chi^Q(\mathbf{Q})/4$ , since  $\chi_l^s(\mathbf{Q}) \gg \chi_{1,1;2,2}^s(\mathbf{Q})$  and  $\chi_l^c(\mathbf{Q}) \approx -\chi_{1,1;2,2}^c(\mathbf{Q})$  near the critical point [26].)  $\Lambda_l^{s,c}$  is the VC for the gap equation, which we call  $\Delta$ -VC in Ref. [27]. The AL-type diagram for the charge channel is given by  $\Lambda_l^c(\mathbf{q}; \mathbf{q}') \sim 1 + T \sum_k \Lambda_{\text{AL}}(\mathbf{q} - \mathbf{q}'; k) G(k) \chi^s(k + \mathbf{q}) \chi^s(k - \mathbf{q}')$ , which is strongly enlarged for  $\mathbf{q} - \mathbf{q}' \approx \mathbf{Q}$ , and the orbital-fluctuation-mediated pairing is favored [25, 27]. The merit of the RG+cRPA method is that the the AL-type  $\Delta$ -VC is automatically produced in calculating the pairing susceptibility in Eq. (1).

In the RPA with  $J > 0$ , the TSC cannot be achieved because of the relation  $\chi_l^s(\mathbf{Q}) \gg \chi_l^c(\mathbf{Q})$  and  $\Lambda^{c,s} = 1$  in the RPA: In this case, spin-fluctuation-mediated SSC is obtained since  $|V_s^l| = (U^2/2)\{3|\Lambda_l^s|^2\chi_l^s - |\Lambda_l^c|^2\chi_l^c\}$  is three times larger than  $|V_t^l| = (U^2/2)\{|\Lambda_l^s|^2\chi_l^s + |\Lambda_l^c|^2\chi_l^c\}$ . In the present RG+cRPA method, in contrast, the relationship  $\chi_l^s(\mathbf{Q}) \sim \chi_l^c(\mathbf{Q})$  is realized, and therefore the triplet interaction  $|V_t^l|$  can be larger than  $|V_s^l|$ . Using Fig. 3 (d), we explain the gap structure of the TSC state induced by orbital+spin fluctuations at  $\mathbf{q} \approx \mathbf{Q}$ . In addition to the necessary nodes shown by solid lines, accidental nodal lines appear around  $k_x \approx \pm\pi/3$  and  $k_x \approx \pm 2\pi/3$ : The reason is that  $\Delta_x^\alpha(\mathbf{q})$  and  $\Delta_x^\beta(\mathbf{q}')$  tend to have the same sign for  $\mathbf{q} - \mathbf{q}' \approx \mathbf{Q}$  due to large attractive interaction by  $V_t^l(\mathbf{q}; \mathbf{q}')$ . For this reason, the relation  $\Delta_x^\beta(\mathbf{q}) \sim \sin 3k_x$  in Fig. 3 (b) is satisfied in the  $E_u$ -type TSC state.

In Fig. 3 (a),  $\chi_s^{\text{SC}}$  also develops at low temperatures: Figures 3 (e) and (f) show the obtained  $A_{1g}$  and  $B_{1g}$  SSC gap structures, which give the first and the second largest  $\chi_s^{\text{SC}}$ 's. Both SSC states with sign reversal are mainly caused by spin fluctuations, and  $A_{1g}$  state is slightly stabilized by the orbital fluctuations. The  $A_{1g}$  state in Fig. 3 (e) dominates the TSC state when  $\chi^s(\mathbf{Q}) \gg \chi^Q(\mathbf{Q})$ , which is realized for  $J/U \gtrsim 0.05$  in Fig. 2 (a).

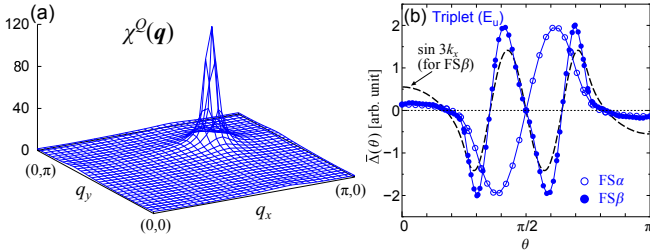


FIG. 4: (color online) Numerical results obtained by the SC-VC method: (a)  $\chi^Q(\mathbf{q})$  and (b) TSC gap function  $\tilde{\Delta}_x^\mu$ .

To verify the reliability of the results given by the RG+cRPA method, we also study the present model using the SC-VC method [25]. Figure 4 (a) shows the obtained  $\chi^Q(\mathbf{q})$  for  $U = 2.33$  and  $J/U = 0.1$  at  $T = 0.05$ . Its peak position at  $\mathbf{q} \approx \mathbf{Q}$  is consistent with the RG+cRPA result in Fig. 1 (d).  $\chi^Q(\mathbf{Q}) = 113$  and  $\chi^s(\mathbf{Q}) = 46$  in the present calculation. By taking the self-energy correction into SC-VC method, the orbital fluctuations will develop even for  $J/U \sim 0.15$  [27]. Next, we can study the superconducting state by solving the linearized gap equation. The obtained largest eigenvalue is  $\lambda_t^{\text{SC}} = 0.495$  ( $E_u$  state) and  $\lambda_s^{\text{SC}} = 0.479$  ( $A_{1g}$  state). The obtained TSC gap function is shown in Fig. 4 (b), which is essentially similar to the gap structure in Fig. 3 (b). Thus, the numerical results of the RG+cRPA method are confirmed by the diagrammatic approach.

The filling of the q1D bands in  $\text{Sr}_2\text{RuO}_4$  is  $n = 2.8$  according to the band calculation [29]. Even in this case, the TSC state with  $\Delta_{x(y)}^\beta \sim \sin 3k_{x(y)}$  is also obtained,

by using both RG+cRPA and SC-VC methods. The obtained peaks of  $\chi^Q(\mathbf{q})$  and  $\chi^s(\mathbf{q})$  coincide and shifts to  $\mathbf{q} \approx (0.6\pi, 0.6\pi)$ .

Even in the RPA, strong orbital fluctuations can be obtained by putting  $U' > U$  [30]. The TSC can be realized by orbital fluctuations as found by Takimoto [13], but the fully-gapped  $A_{1g}$  state is also a natural candidate. Within the RPA, the SSC state is obtained for any  $J = (U - U')/2$ , and fully-gapped  $A_{1g}$  appears for largely negative  $J$ . To obtain the TSC within the RPA, we have to choose the ratios  $U'/U > 1$  and  $J/U$  independently to maintain the coexistence of orbital and spin fluctuations. In contrast, in the RG+cRPA method, both fluctuations coexist due to the orbital-spin mode-coupling, and the TSC is obtained for a wide range of parameters under the condition  $J = (U - U')/2 > 0$ .

When the TSC occurs in the q1D FSs in real compound, the superconducting gap on FS $\gamma$  will be induced from q1D FSs (proximity effect), due to weak inter-band electron correlation in addition to the large SOI of 4d-electron. As for the latter effect, large orbital mixture between FS $\beta$  and FS $\gamma$  due to the SOI is predicated by the first-principle study [29]. It is an important future problem to study the TSC in three-orbital model for  $\text{Sr}_2\text{RuO}_4$ , by taking the SOI into account. The  $\mathbf{d}$ -vector [31, 32] and the topological properties of the TSC state [33–36] can be discussed by this study.

In summary, we proposed the orbital+spin fluctuation-mediated TSC in  $\text{Sr}_2\text{RuO}_4$  by analyzing the two-orbital Hubbard model using the RG+cRPA method. Thanks to the VC neglected in the RPA, strong orbital and spin fluctuations at  $\mathbf{q} \sim \mathbf{Q}$  emerge in the q1D FSs. The TSC is obtained for  $J/U \lesssim 0.04$  (0.08) without (with) the cVC for  $\Lambda_0 = 1$ . Similar TSC gap structure is obtained by the SC-VC method for  $J/U \lesssim 0.1$ . The present work demonstrated that the RG+cRPA method is very powerful in the study of various 2D strongly correlated systems, emergence of orbital/spin order and superconductivity.

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